EXERCISE – V HINTS & SOLUTIONS

Sol.1 $I = \int \sin^{-1}\left(\frac{2x+2}{\sqrt{4x^2+8x+13}}\right)dx$

put $2x + 2 = 3\tan\theta \Rightarrow 2dx = 3\sec^2\theta\, d\theta$

$= \int \sin^{-1}\left(\frac{3\tan\theta}{3\sec\theta}\right)\frac{3}{2}\sec^2\theta\, d\theta$

$= \frac{3}{2}\int \sin^{-1}(\sin\theta)\sec^2\theta\, d\theta = \frac{3}{2}\int \theta\sec^2\theta\, d\theta$

$= \frac{3}{2}\left[\theta\tan\theta - \int \tan\theta\, d\theta\right] = \frac{3}{2}[\theta\tan\theta - \ell n\sec\theta]$

where $\tan\theta = \frac{2x+2}{3}$ & $\sec\theta = \frac{1}{3}\sqrt{4x^2+8x+13}$

Sol.2 $I = \int (x^{3m}+x^{2m}+x^m).(2x^{2m}+3x^m+6)^{1/m}dx$

$= \int (x^{3m}+x^{2m}+x^m)\frac{(2x^{3m}+3x^{2m}+6x^m)^{1/m}}{x}dx$

$= \int (x^{3m-1}+x^{2m-1}+x^{m-1})(2x^{3m}+3m^{2m}+6x^m)^{1/m}dx$

put $2x^{3m} + 3x^{2m} + 6x^m = t$

$6m(x^{3m-1} + x^{2m-1} + x^{m-1})\,dx = dt$

$I = \int \frac{t^{1/m}dt}{6m} = \frac{1}{6m}\frac{t^{\frac{1+m}{m}}}{\frac{1+m}{m}} + c$

$= \frac{1}{6(1+m)}(2x^{3m} + 3x^{2m} + 6x^m)^{\frac{m+1}{m}} + c$

Sol.3 $I = \int \frac{x^2-1}{x^3\sqrt{2x^4-2x^2+1}}dx = \int \frac{\left(\frac{1}{x^3}-\frac{1}{x^5}\right)}{\sqrt{2-\frac{2}{x^2}+\frac{1}{x^4}}}dx$

Let $2 - \frac{2}{x^2} + \frac{1}{x^4} = t^2 \Rightarrow \left(\frac{4}{x^3}-\frac{4}{x^5}\right)dx = 2t\,dt$

$= \int \frac{t\,dt}{2t} = \frac{1}{2}t + c = \frac{1}{2}\sqrt{2-\frac{2}{x^2}+\frac{1}{x^4}} + c$

$= \frac{\sqrt{2x^4-2x^2+1}}{2x^2} + c$

Sol.4 **(a)** Here $f(x) = \frac{x}{(1+x^n)^{1/n}}$

$fof(x) = \frac{f(x)}{[1+f(x)^n]^{1/n}} = \frac{x}{(1+2x^n)^{1/n}}$

$fofof(x) = \frac{x}{(1+3x^n)^{1/n}}$

$\Rightarrow g(x) = \underbrace{(fofo......of)}_{n\ terms}(x) = \frac{x}{(1+nx^n)^{1/n}}$

Hence $I = \int x^{n-2}g(x)\,dx = \int \frac{x^{n-1}}{(1+nx^n)^{1/n}}dx$

$= \frac{1}{n^2}\int \frac{n^2x^{n-1}dx}{(1+nx^n)^{1/n}} = \frac{1}{n^2}\int \frac{\frac{d}{dx}(1+nx^n)}{(1+nx^n)^{1/n}}dx$

$= \frac{1}{n(n-1)}(1 + nx^n)^{1-1/n} + c$

(b) $F(x) = \int \sin^2 x\,dx = \int \frac{1-\cos 2x}{2}dx$

$f(x) = \frac{1}{4}(2x - \sin 2x) + c$

since $F(x + \pi) \neq F(x)$

Hence statement–1 is false

but statement–2 is true as $\sin^2x$ is periodic with period π

Sol.5 $J - I = \int \frac{e^x(e^{2x}-1)}{e^{4x}+e^{2x}+1}dx$

put $z = e^x \Rightarrow dz = e^x\,dx = \int \frac{z^2-1}{(z^4+z^2+1)}dz$

$= \int \frac{1-\frac{1}{z^2}}{\left(z+\frac{1}{z}\right)^2-1}dz = \frac{1}{2}\ell n\left(\frac{e^x+e^{-x}-1}{e^x+e^{-x}+1}\right) + c$

$J - I = \frac{1}{2}\ell n\left(\frac{e^{2x}-e^x+1}{e^{2x}+e^x+1}\right) + c$

Sol.6 **C**

$\tan x = t$

$\Rightarrow \sec^2x\,dx = dt$

$\int \frac{dt}{(t+\sqrt{1+t^2})^{9/2}}$ put $t + \sqrt{1+t^2} = z$

$= \frac{1}{2}\int \frac{z^2+1}{z^{13/2}}dz = \frac{-1}{7}\frac{1}{z^{7/2}} - \frac{1}{11}\frac{1}{z^{11/2}} + c$

$= -\frac{1}{(\sec x+\tan x)^{11/2}}\left\{\frac{1}{11}+\frac{1}{7}(\sec x+\tan x)^2\right\} + c$